2. **[10]** In this question, your task is to find **integer solutions** to the following equation.



where *x*1*,x*2*,...,x*6 are unknowns, and *A,B,C,D,E,F,S* are given constants, all these constants are integers and can be either positive or negative.

We want to use a computer algorithm to find **all integer solutions** to the above equation, each of which satisfies the following condition:

1 ≤ *xi* ≤ *N* ∀*i* ∈{1*,*2*,*3*,*4*,*5*,*6}

where *N* is a given constant, which is typically a large positive integer.

A na¨ıve algorithm would be to enumerate all possible 6-tuples of (*x*1 *...x*6) and check each tuple to see whether it satisfies the equation. Clearly, this algorithm takes O(*N*6) time to run and is way too slow for large values of *N*. So we would like to devise a faster algorithm with the help of some data structure we learned. The asymptotic runtime of your algorithm should be orders of magnitude lower than O(*N*6), for example, O(*N*5), O(*N*3), O(*N*2) or O(log*N*).

Design an algorithm that is **as fast as possible**, write down a detailed and clear description of the algorithm (what data structure is used and how the algorithm utilizes the data structure). Provide the **pseudocode** of your algorithm, and justify its **correctness** and **runtime**. Only the fastest possible algorithm with clear justification will get full marks.

**Notes/Hints:**

* Consider using a hash table, and you may assume that hash table supports worst-case O(1) runtime for the Search, Insert and Delete operations. Do NOT repeat the details of the hash table operations that have been discussed in the lecture.
* Do NOT try to do anything mathematically sophisticated. Our attitude here is to just write some code and let the computer do the work of searching for solutions.